Finite Element Mesh Partitioning based on Multigraph Diffusion

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FEM Simulations:

- Simulation space is discretized into a mesh.
- Mesh is partitioned and distributed to the processing nodes.
- All processing nodes should have same amount of work.
- Communication is expensive. Small partition boundary is wanted!
- After mesh changes, partitions have to be rebalanced.
Existing libraries (Chaco, Metis, Jostle, Party) follow the multi-level paradigm.

- Smaller, similar instances are created via a matching.
- Smallest Graph is partitioned.
- Solution is mapped onto larger graphs and improved by vertex exchange heuristics.

Two important operations:
- Matching Algorithm (HEM, LAM).
- Vertex Exchange Heuristic (KL, HS).
**Motivation**

State-of-the-art heuristics:

- are fast.
- compute solutions with low cuts.
- mainly minimize cut, but not real communication costs (B. Hendrickson).
- cause relatively high migration (L. Oliker).
- are based on sequential vertex exchange strategies.
- do not compute smooth partition boundaries.
- cannot even guarantee connectivity.

**Idea:** Round partitions with smooth borders should cause little communication (at least for dual graphs from FEM computations).
Example (100 × 100 Grid)
Advantages of shape optimized partitions

**Jostle**
- cut: 549
- boundary: 992
- comm: 1020

**Shape optimized partitioning**
- cut: 578
- boundary: 862
- comm: 892
The Bubble Framework
A shape optimizing learning framework

Consists out of three operations:

1. Determination of initial seeds.
2. Partition growing.
3. Determination of partition centers (seeds).

Init, Grow and Move
The First Order Diffusion Scheme

FOS is a local iterative algorithm.

- Initial load diffuses into a graph.
- Converges towards $\| \cdot \|_2$-minimal balancing flow.

### Definition FOS

Given a connected graph $G = (V, E)$, a suitable constant $\alpha$ and the initial load vector $l$, the local iterative *First Order Scheme* computes a $l_2$-minimal balancing flow $f$. In each iteration $i$, it performs:

\[
\begin{align*}
x^i_{e=(u,v)} &= 1/\alpha \cdot (l^i_u - l^i_v) & (1) \\
f^{i+1}_e &= f^i_e + x^i_e & (2) \\
l^{i+1}_v &= l^i_v - \sum_{e=(v,*) \in E} x^i_e & (3)
\end{align*}
\]
A New Growing Mechanism
Integrating Diffusion into the Bubble Framework

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The Bubble Framework
Diffusion in Graphs
Integration

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Conclusion
Decrease Run-Time by Levels
Adopting the Multilevel Paradigm

Two possible ways:
- Decrease the problem size.
- Faster Diffusion.

Combine them in the following way:
- Create a level hierarchy and choose two designated levels.
- Learn on the middle level.
- Compute the diffusion with the help if the lower levels.
- Finally interpolate to the highest (initial) level.

Dedicated Levels and Possible Transitions
Diffusion on the lower and its projection onto the middle level.
Conclusion and Further Work

Applying Levels:
- Run-time decreases by about one order of magnitude.
- Speed-up less than expected.
- Solution quality does not decrease, if right parameters are chosen.

But:
- The choice of the iteration number becomes more difficult.
- Resulting Algorithm is less reliable.

Further (meanwhile finished) Work: Problem can be solved by applying an alternative diffusion scheme (FOS/A).