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Paderborn, May, 4th 1:00 pm
Return until May, 11th 1:00 pm
to office F2.401

# Home work for
# **Fundamental Algorithms**
## SS 2007
## Sheet 4

**Exercise 10:** Show that the edge weights $\tilde{w}$ as used in Johnson's algorithm are nonnegative.

**Exercise 11:** For a directed graph $G = (V, E)$ the transitive closure $G^* = (V^*, E^*)$ is defined by

$$\begin{aligned} V^* &= V \\ E^* &= \{(u, v) \mid u \leadsto_G v\} \end{aligned}$$

Develop an algorithm to compute, given a directed graph $G$, the transitive closure $G^*$ of $G$ in time $O(n^3)$.

**Exercise 12:** (*)
A **system of difference constraints** is given as $Ax \leq b$ where $A \in \{-1, 0, 1\}^{m \times n}$, $b \in \mathbb{Z}^m$ and $x$ is a vector of $n$ unknowns to be determined. Moreover, each row of $A$ contains exactly one 1 and one -1, all other entries are 0.
For a system $Ax \leq b$, $A \in \{-1, 0, 1\}^{m \times n}$, $b \in \mathbb{Z}^m$ of difference constraints the **constraint graph** $G = (V, E)$ is given as

$$\begin{aligned} V &= \{v_0, v_1, \ldots, v_n\} \\ E &= \{(v_i, v_j) \mid \exists k : A_{kj} = 1 \text{ und } A_{ki} = -1\} \cup \{(v_0, v_i) \mid 1 \leq i \leq n\} \end{aligned}$$

For the constraint graph $G$ an edge weight function $w$ is given as $w(v_0, v_i) = 0$, $1 \leq i \leq n$ and $w(v_i, v_j) = b_k$ where $k$ is the row of $A$ corresponding to the edge $(v_i, v_j)$.
Prove:

a) If the constraint graph $G$ does not contain any negative weight cycle then $x = (\delta(v_0, v_1), \ldots, \delta(v_0, v_n))$ is a feasible solution of $Ax \leq b$.

b) If $G$ contains a negative weight cycle then $Ax \leq b$ does not have any solution.