

Home work for
Fundamental Algorithms
SS 2007
Sheet 2

Exercise 4: Let $G = (V, E)$ be a directed graph, let $C_1, \dots, C_k \subset V$ denote the SCCs of G . The **component graph** $G_C = (V_C, E_C)$ of G is defined by

$$V_C = \{C_1, \dots, C_k\}$$

$$E_C = \{(C, C') \mid \exists u \in C, v \in C', (u, v) \in E\}$$

Show that the component graph G_C of any directed graph G is a DAG.

Exercise 5: Let $G = (V, E)$, $|V| = n$, be a directed graph with weight function $w : E \rightarrow \mathbb{R}$. Let $s \in V$ and let G be initialized by INITSSSP(G, s).

a) Suppose G does not contain any negative weight cycle.

Prove that there is a sequence of $n - 1$ executions of RELAX such that after execution of this sequence $d[v] = \delta(s, v)$ for all $v \in V$.

b) Now let G contain a negative weight cycle reachable from s .

Show that an infinite sequence of executions of RELAX can be constructed such that every relaxation causes a shortest path estimate to change.