

Home work for  
**Fundamental Algorithms**  
SS 2007  
Sheet 1

**Exercise 1:** Prove or disprove:

If there is a path from a node  $u$  to a node  $v$  in a directed graph  $G$  such that after a depth first search on  $G$   $d[u] < d[v]$ , then  $v$  is a descendant of  $u$  in the DFS forest produced.

**Exercise 2:** A vertex cover of an undirected graph  $G = (V, E)$  is a subset  $C \subset V$  such that for each  $\{u, v\} \in E$  we have  $u \in C$  or  $v \in C$ .

- a) Develop an efficient algorithm that, for a given undirected connected graph  $G$ , outputs a vertex cover  $N$  for  $G$ .
- b) Let  $G$  be any undirected connected graph, let  $N$  be the vertex cover computed by your algorithm from a). Let  $C$  be any vertex cover of  $G$ .  
Either show that  $|N| \leq 2 \cdot |C|$  or give a counterexample.

**Exercise 3:** (Programming task: to be returned by email until April 27th)

Write a program that

- a) reads a directed graph from an input file, with the input specification as given below,
- b) represents the graph internally by adjacency lists,
- c) checks using DFS whether the graph is cycle free,
- d) checks using DFS whether the graph is strongly connected,
- d) outputs the result according to the output specification given below.

To complete this task you must send by email to [obelix@upb.de](mailto:obelix@upb.de)

- 1) The source code of your program
- 2) The command to compile your program (this must be possible on computer [mistel.cs.upb.de](http://mistel.cs.upb.de) (Linux system))
- 3) The command to start your program. The input file name must be given as argument

Input specification:

<comment line>\*

n = <int>

m = <int>

<int> : <list of int>

where <comment line> is a line starting with #, and <list of int> is a list of nonnegative integers separated by a blank (the adjacency lists). An example is given below:

```
# This is a graph with 3 nodes, 5 edges
```

```
n = 3
```

```
m = 5
```

```
0 : 1 2
```

```
1 : 2
```

```
2 : 1 0
```

Output specification:

Output should consist of exactly two lines of the form  
cycle-free < {0,1} >

s-connected < {0,1} >

where 0 means that the graph is not cycle-free (or strongly connected) and 1 means that it is. An example is given below:

```
cycle-free 0
```

```
s-connected 1
```