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Composition of Efficient Nested BSP Algorithms: Minimum Spanning Tree Computation as an Instructive Example*

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Abstract

We report on the results of an automatic configuration approach for implementing complex parallel BSP algorithms. For this approach, a parallel algorithm is described by a sequence of instructions and of subproblems that have to be solved by other parallel algorithms called as subroutines, together with a mathematical description of its own running time. There also may be free algorithmic parameters as, e. g., the degree of trees in used data structures that have an impact on the running time. As the running time of an algorithm depends on several machine parameters, on some fixed and on the choice of the free algorithmic parameters and on the choice of the parallel subroutines for which the same statement applies in turn, the actual composition of the parallel program for an actual parallel machine from all these ingredients is a difficult task. We have implemented such a configuration system using the Paderborn University BSP library and present as an instructive example the theoretical and experimental results of implementations of sophisticated minimum spanning tree algorithms.

1 Introduction

Motivation. The actual running time of implementations of parallel algorithms depends on two groups of parameters, namely "software" parameters such as, e. g., the number of memory accesses or the degree of a used broadcast tree, and "hardware" parameters such as, e. g., the time necessary to set up a communication or the number of available processors. This leads to the observation that for different parameter constellations different "plain" algorithms are the fastest ones. Furthermore, sophisticated parallel algorithms often make recursive calls and call subroutines that have similar dependencies as mentioned above and that themselves solve complex subproblems for which clever algorithms have been designed and implemented. E. g., efficient parallel algorithms for minimum spanning tree computation use parallel sorting algorithms, broadcast methods and make recursive calls.

Hence, in order to have efficient parallel programs for an actual machine, they have to be *configured*. That means that one has to decide which parallel algorithm has to be taken and which subroutines have to be used on what portion of the parallel machine depending on both types of parameters.

For the Bulk Synchronous Parallel (BSP) model and its extensions, a large variety of efficient parallel algorithms for many problems has been developed and quite accurately analyzed. There are BSP environments as, e. g., the Paderborn University BSP library (PUB) [4] and the Oxford BSPlib [9] that provide a platform that can efficiently execute BSP-like programs. This enables the programmer to choose for the composition of the final program from a pool of available algorithms for his or her parallel machines. However, configuring a parallel program becomes quickly very complex due to the various parameters and (possibly mutually influencing) dependencies. Therefore, it should be done automatically by a program we call *configurator*.

Our contribution. We have implemented a comparatively simple prototype of a configurator that, for a given problem Π, calculates from a library of descriptions of BSP algorithms for Π a composite program for a given machine and input size. All algorithms are written for an extension of Valiant's Bulk-Synchronous Parallel model (BSP, [13]) using the C library PUB ([4]) that supports all functions for the implementation of BSP programs with subalgorithms. To evaluate the algorithms and to be able to predict the running time, we appropriately adapt the BSP model. The composite program obtained by the configurator is always optimal for this BSP extension.

In order to verify the soundness of the configuration approach and the BSP extension, we present the results of implementations of various algorithms for minimum spanning tree (MST) computation and compare their running times with the measured and predicted running time of the program that is output by the configurator. Our MST algorithms use complex subroutines as, e. g., different integer

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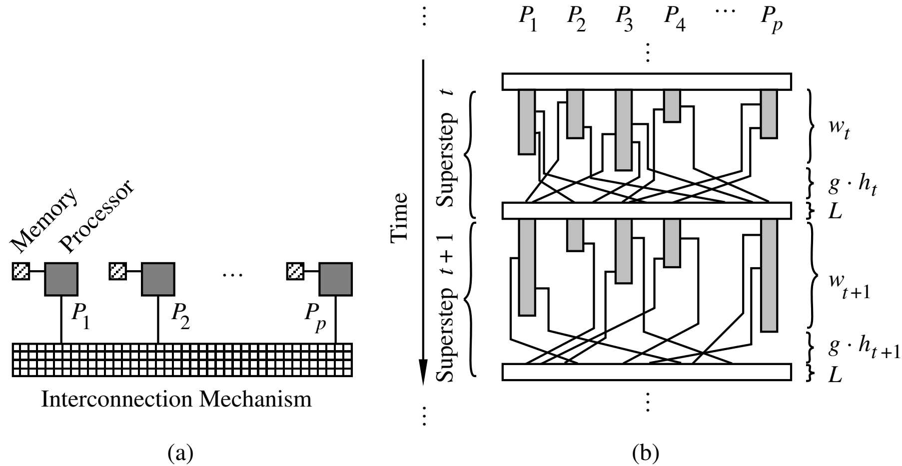


Figure 1. The BSP model. (a) The BSP machine. (b) Execution of a BSP algorithm.

sorting routines, broadcast methods, prefix operations, etc., which are in turn also subject to the configuration. For a full overview of the bundle of problems and algorithms we have investigated and implemented, see the "map" provided in Fig. 6. Rather than inventing new MST algorithms or to devise a fast configurator, the aim of this paper is to present what can be achieved by the composition of an efficient parallel MST program from the existing MST algorithms (and the necessary subroutines). Further results on automatically configured programs for the broadcast problem can be found in [3].

Organization of paper. The paper is organized as follows: In Section 2, we give a short overview of the used extensions of the BSP model, the PUB library, and the parallel computer on which we performed our experiments. In Section 3, we specify the input, output and used library and algorithm of our prototypical configurator. In Sections 4 and 5, we present the implementation details and discuss the running times of a collection of minimum spanning tree algorithms.

2 The BSP Model and the Used Platforms

The BSP model. Now we give a short sketch of the BSP model. A discussion of this and other parallel bridging models and their advantages and disadvantages can be found in [10]. In Valiant's Bulk Synchronous Parallel (BSP) model [13], a parallel computer consists of a set of processors with local memory, an interconnection mechanism that allows point-to-point communication, and a mechanism for barrier-style synchronizations (see Figure 1(a)). This is the so-called BSP machine. The hardware parameters are p, the number of available processors, g, the bandwidth inefficiency, i. e., the (normalized) time necessary to hand over one Byte to the interconnection mechanism, sometimes also called the *gap*, and L, the time the machine needs for the barrier synchronization. For concrete machines, g and L have to be determined experimentally.

The execution of an BSP algorithm is a sequence of *supersteps*, each terminated by a barrier synchronization. In every superstep, all processors do local computations and send messages to other processors. These messages are not available until the next synchronization has been finished (see Figure 1(b)). The cost, i. e., the running time, of the t-th superstep on a BSP machine is given by $w_t + g \cdot h_t + L$, where, for superstep t, w_t is the maximum local work performed by any processor and h_t is the maximum number of Bytes sent or received by any processor. After devising a BSP algorithm, the total amount of local work, the total communication volume and the number of supersteps have to be determined by careful analysis.

A variant of the BSP cost model that rewards block-wise communication is BSP* introduced in [2]. It has an additional machine parameter, the critical block size B. In this extension, the communication cost of a single packet to be sent is $\max\{s, B\}$ where s denotes the number of Bytes of the packet. The maximum accumulated communication cost incurred in any processor during a superstep replaces the number of Bytes from the plain BSP model. Note that g has to be adapted accordingly.

Another extension is the *decomposable* BSP model (D-BSP) [6]. Here the BSP machine may be divided into several partitions, each acts like an independent BSP machine of smaller size. The BSP* parameters B, L and g are now functions of the partitions' size. Hence, algorithms for the D-BSP model can exploit locality. Moreover the partitions can execute different algorithms independently.

The impact of these extension on algorithm design is discussed in [11].

Table 1. The BSP* parameters of the Pentium III workstation cluster.

(a) MPI with SCI (2D torus)

p	B [B]	L [μs]	g [ns^{-1}]
2	316	2.10	32.4
4	509	4.49	37.5
8	389	6.28	43.9
16	21	9.08	194.5

(b) TCP/IP with Fast Ethernet (complete graph)

p	B [B]	L [μs]	g [ns^{-1}]
2	3092	98.9	117.1
4	1467	145.5	121.8
8	264	140.7	629.2
16	239	152.7	617.9

The PUB library. All implementations use the Paderborn University BSP library PUB. A detailed description of PUB can be found in [4] and, comprehensively, in [12]. PUB is a C library that supports the development and implementation of parallel algorithms designed for the BSP model. It incorporates the use of block-wise communication as suggested by BSP*, and it provides dynamic partition of the machine as suggested by D-BSP.

The used parallel machines. For all experiments, we used a cluster of 96 Linux workstations with two different communication mechanisms. The cluster is operated by the Paderborn Center for Parallel Computer (PC^2). Every workstation has two Pentium III processors with 850 MHz clock rate and 512 MB memory. As communication mechanism, there are two alternatives: One can choose to use a 2-dimensional torus of SCI links as interconnection network, and one can decide to choose a Fast Ethernet with a Cisco Catalyst 5509 switch (and, hence, a complete graph as interconnection network). For the communication, MPI and TCP/IP are used, resp.

Table 1 presents the BSP* parameters of these two parallel computers. They were used by our configurator in the computation of the cost of our implemented algorithms. A compilation of the BSP* parameters of further machines as the Cray T3E and Parsytec CC can be found in [12].

3 The Configurator: Input/Output Specification and Algorithm

In what follows, we define how to describe algorithms and introduce the term *schedule* for a given problem. This is the necessary adaptation of the BSP model. A schedule fixes the algorithms and all free parameters to be used to solve a problem and all occurring subproblems. The input of the configurator is the name Π of the problem, the name of the machine (i. e., which BSP parameters apply), and a specification of the input of Π. The configurator works on a library of algorithm descriptions. It outputs the schedule. This schedule is used during the execution to determine the real (sub-)program that will be executed. It is the equivalent to the actual parallel program.

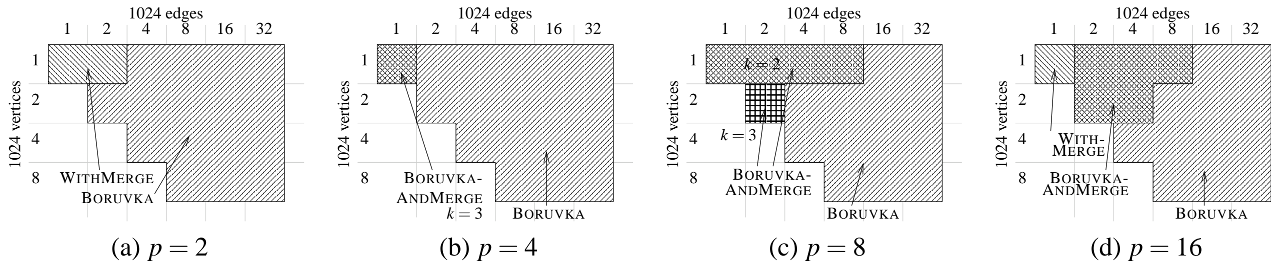
An *algorithm description* A consists of the following five components: (1) Π is the name of the problem the algorithm solves. (2) p_{count} is the set of feasible machine sizes, i. e., the machine sizes for which the algorithm A can work depending on the input for A. (3) v_{count} is the number of different possible choices for fixing the free algorithmic parameters of A. E. g., in a broadcast algorithm this might be the number of feasible tree degrees. (4) t is the function that computes, depending on a given algorithmic variant, the running time of A without the time A will spent in subcalls. Note that in the algorithm description, it is not known in advance which subcalls will be used in the schedule. (5) r is an indexed list of the possible variants that can constitute the algorithm. Depending on the index, r returns the name of a (sub-)problem to be solved, the input size and the number of involved processors that together describe this single variant. A more formal specification of the components with all details on the parameters involved can be found in [3].

We say that A *solves* problem Π.

Given a set $\mathcal{P}$ of problems and a set $\mathcal{A}$ of algorithm descriptions, a *valid schedule* S for a problem $\Pi \in \mathcal{P}$ and its input description is defined recursively. An algorithm $A \in \mathcal{A}$ solving $\Pi \in \mathcal{P}$ is fixed, as well as all free parameters, and there are valid schedules for all occurring subproblems $\Pi'_i \in \mathcal{P}$. The recursion terminates when there are no further subproblems. So a valid schedule S can be viewed as a *schedule tree* directed from the root to the leaves.

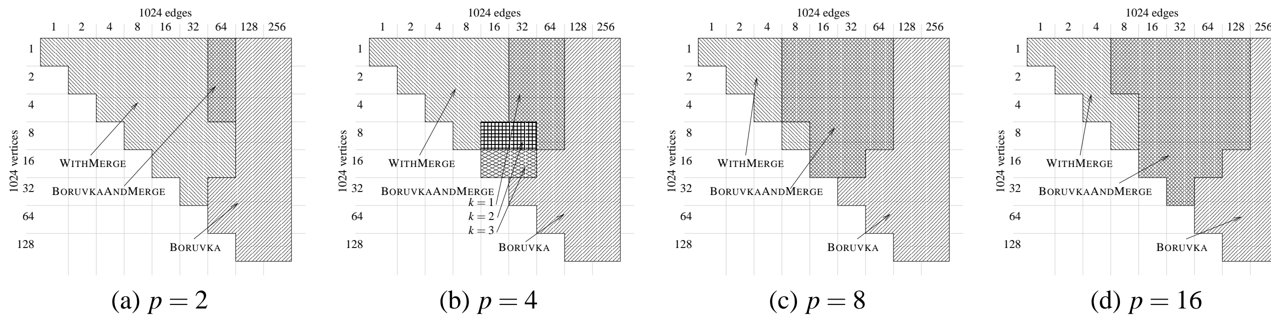
Let S be a valid schedule for a problem Π that has to be executed on a p processor BSP machine given by its machine parameters. The *cost* of S, i. e., its (predicted) running time on the BSP machine, is defined along the schedule tree. The cost of the root is the cost of A (without the subroutine calls) given by t (see point (4) above) plus the sum of the cost of all children of the root.

We have implemented a prototypical configurator that computes a schedule tree (in a bottom-up way) and, hence, a valid schedule with minimum cost by a brute force search testing all possible valid schedules. Note that this computation is offline, i. e., it is done only once before the schedule is used for many same-sized inputs. The configurator works for arbitrary problems Π and algorithm descriptions A. The running time of the configurator was, for our MST experiments, just a few minutes.



(a) $p = 2$ (b) $p = 4$ (c) $p = 8$ (d) $p = 16$

Figure 2. Results of the configuration on a Pentium III workstation cluster with SCI.



(a) $p = 2$ (b) $p = 4$ (c) $p = 8$ (d) $p = 16$

Figure 3. Results of the configuration on a Pentium III workstation cluster with Fast Ethernet.

4 An Example: Parallel Minimum Spanning Tree Algorithms

As example for a complex problem with a rich combinatorial structure, a usually irregular communication pattern, and a variety of sophisticated algorithms that in turn use clever subalgorithms, we use the problem of computing a minimum spanning tree (MST) for an undirected weighted graph. For the algorithmic background of the MST problem, see [5, Chap. 24]. In this section, we give some notes on the implemented algorithms. In the next section, we report on the experiments by running the composite programs and the plain programs, and we compare their actual running times and relate them to the cost computed by the configurator.

Each of our implemented algorithms is based on the following three basic operations:

(a) Operation “Kruskal”: Kruskal’s sequential algorithm (e. g., see [5, Sec. 24.2]) tests for every edge, in order of increasing weights, whether it can be included in the minimum spanning tree, i. e., whether it connects two connected components (called *supervertices*) created by the edges chosen so far.

(b) Operation “Borůvka step”: In a Borůvka step (for a nice and detailed description, see [8]) every supervertex selects its cheapest outgoing edge. These edges are added to the MST edges, avoiding cycles (by construction, these cycles are not longer than 2). After that, the new supervertices, i. e., the connected components, are calculated, the edges are relabeled according to the new supervertices, and all edges belonging to the same supervertex are removed. This step reduces the number of vertices by a factor of at least two.

We have implemented two different solutions for this problem, namely DenseBoruvkaStep which is step (2) of algorithm Mst-Dense in [1], and BoruvkaStep which is in essence from [7, 8].

DenseBoruvkaStep is specially designed for dense graphs. It calculates the lightest edge of all locally stored edges and then executes a parallel prefix operation to determine the edges with global minimum weights, for every supervertex.

BoruvkaStep creates adjacency lists for all vertices by grouping edges of the same vertices by integer sorting. Then the minima of each group are calculated by a parallel segmented prefix operation (see [8], Section 4.1.4, for details). Our implementation uses a sequential algorithm for computing the connected components.

(c) Operation “MstMerge”: This operation (Step (2) of Mst-Merge in [1]) merges local MSTs. It uses a d-ary communication tree. d is a free parameter to be set by the configurator. Each tree node sends its MST to its predecessor, the predecessors merge the MSTs by calculating an MST of all edges received. In the end, the root of the communication tree knows the global minimum spanning tree.

Figure 6 in the appendix shows a full map of all used algorithms, the subproblems and their relationships. In the following, we give short remarks on the three parallel MST algorithms that can be found in the map. I. e., the algorithmic description of our general solution of the MST problem has three main variants: MstWithMerge, MstBoruvka, and MstBoruvkaAndMerge.

MSTWITHMERGE: This algorithm solves the MST problem by calculating the local spanning tree and merges all these trees with operation MSTMerge. Then the result is distributed from the root to all nodes.

MSTBORUVKA: This algorithm executes Borůvka steps until the number of supervertices is 1. Since each step reduces the number of supervertices by at least a factor of 2, at most $\lceil \log_2 n \rceil$ of these steps have to be repeated (with n denoting the number of vertices).

MSTBORUVKAANDMERGE: This algorithm combines Borůvka steps and merging. First it executes some number k of Borůvka steps in order to reduce the number of vertices, then it calculates the minimum spanning tree using MSTMerge. The number k of Borůvka steps is a free parameter of the algorithm and has to be set by the configurator.

Note that Figure 6 even contains cycles that are caused by recursive calls. Of course, the configuration terminates because the parameters usually decrease.

5 Experimental Evaluation of the MST Implementations

5.1 Configured Programs: the Schedules

Figure 2 shows as the result of the configurator the selected algorithms and the fixing of the free parameters for different sizes of the input graphs and different number of processors for the parallel machine interconnected as a 2-dimensional torus of SCI links.

On a computer with a fast network, the algorithm MSTWITHMERGE is chosen on small graphs only. This algorithm has a small number of supersteps, namely $\log_d p$, but calculates a minimum spanning tree sequentially on $(n-1)d$ edges for n vertices in every node of the d-ary communication tree. Also, most of the processors (i. e., $p - p/d^{i-1}$) are idle in round i. If the network is slow (as it is the case with Fast Ethernet), the change from one chosen algorithm to another occurs for larger graph sizes, as can be seen in Figure 3 that presents the results of the automatic configuration for the cluster with Fast Ethernet.

5.2 Measured Running Times

Due to the limited space, we only present the results of two significant series of measurements, one for each communication mechanism. For more measurements, see [3].

Figure 4 shows the results of the measurements on the workstation cluster with $p = 16$ processors. Our graphs consist of $n = 2048$ vertices and randomly chosen edges. Figure 4(a) shows the running times, (b) the ratio of the measurements and the BSP cost, i. e., the predicted times, for the SCI case. The divergence for the algorithms that use the operation MSTMerge is due to inaccurate predictions of the local work. In merging-based algorithms, the sequentially merging of minimum spanning trees dominates the running time. The model counts the local memory accesses of these operations. However, the algorithms run much faster than predicted due to cache effects.

The configured program results, as Figure 4 shows, in the best running time if the input graphs are dense. Otherwise, it comes close to the fastest algorithm. The peaks that can be observed for the configured program in both parts of the figure at 8192 edges is due to the fact that the configurator prefers MSTBORUVKA to MSTBORUVKAANDMERGE too early.

Figure 5 presents the measurements and the prediction accuracy ratio for the Fast Ethernet communication on $p = 16$ processors and graphs with $n = 8192$ vertices. The configurator does not choose the best variant which is mostly MSTWITHMERGE because with Fast Ethernet the network load being a dominating parameter in the running time is not sufficiently covered by BSP. More specifically, if the input graph is dense, during a broadcast only one processor sends data, all other processor receive data. So there is no high network load, and the gap g, listed in Table 1 and measured under high load, is inappropriately large. So the running time is considerably overestimated by the configurator.

6 Concluding Remarks

In this paper, we have presented the configuration approach to obtain fast parallel BSP programs for solving algorithmically complex problems. As a case study, we have presented the results of applying the configuration approach to the minimum spanning tree problem.

In [3], a case study for the broadcast problem can be found. Further problems that are planned to be approached by automatic configuration are the computation of powers in finite fields, and the Independent Set Problem.

Furthermore, the development of faster configurators and the development of configurators that work online are conceivable.

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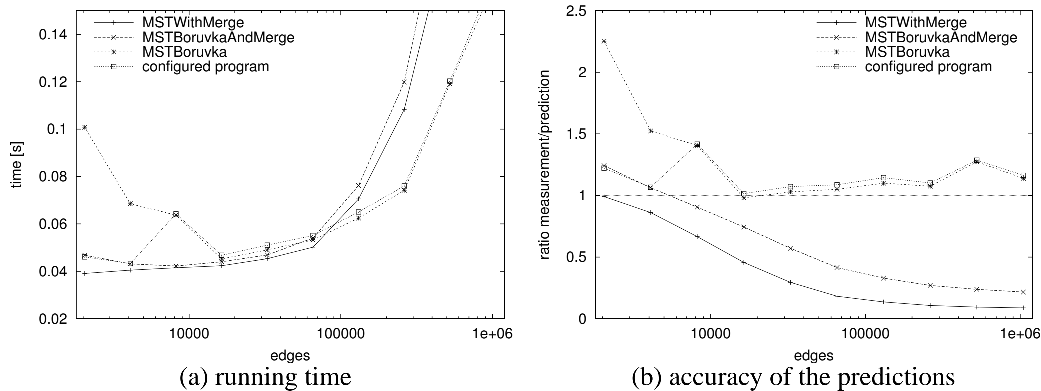
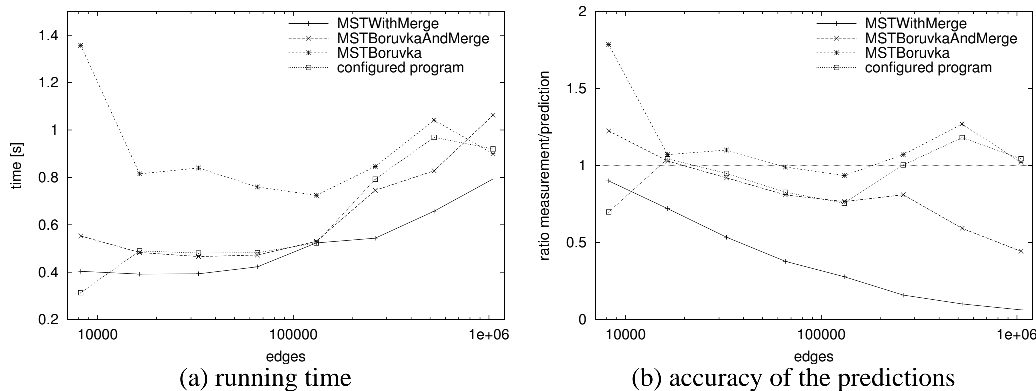


(a) running time (b) accuracy of the predictions

Figure 4. Results for random graphs with n=2048 vertices on p=16 processors, SCI.



(a) running time (b) accuracy of the predictions

Figure 5. Results for random graphs with n=8192 vertices on p=16 processors, TCP/IP.

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Appendix

MST
KRUSKAL
MSTBORUVKA
MSTBORUVKAANDMERGE
MSTWITHMERGE
MSTMerge
ScanAddInteger
MERGESORT
BoruvkaStep
INTEGERADDTREEPREFIX
MSTMERGE
Sort
BORUVKASTEP
DENSEBORUVKASTEP
EdgeMinimum
EDGEMINIMUMTREEPREFIX
SegmentMinimum
SegmentMinimum
Broadcast
TREEBROADCAST
IntegerSort
INTEGERSORTWITHSORT
ROOTBROADCAST
ScanAllAddInteger
DISTRIBUTEDSORTWITHMASTER
SCATTERGATHERBROADCAST
INTEGERWITHMASTERSORT
RADIXSORT
SEQRADIXSORT
problem
solved by
creates subproblem
ALGORITHM

Figure 6. Map of the problems and algorithms for the MST problem